Linear Models, Data, and Geometry

DRAFT material

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Introduction

Even though many schools teach geometry and algebra in different courses, there are many situations in which they connect, largely through formulas that are about some sort of spatial phenomenon. For example, in a math book, you might see:

> A 5-meter ladder is leaning against a wall. The base of the ladder is 1 meter from the base of the wall. How high is the top of the ladder?

The point of this problem is to get you to use the Pythagorean Theorem. It demands that you combine geometry and algebra: geometry to recognize a right triangle and recall the appropriate formula, and algebra to solve that formula for the relevant side.

The activities in this booklet will give you practice making these connections. And they will do it in a special way: through *data*. You won't calculate the height of the ladder, you'll *measure* it. Here's the basic idea:

1. We present you with a situation with some geometry in it.

2. You take some measurements, record them in a data table, and plot them on a graph.

3. You figure out a mathematical function that has the same pattern as the data (this is a *mathematical model* of your data).

This last step is the critical one, of course. As you will see, there are two ways this can work:

- You use your understanding of the geometry and the situation to come up with your model.

- You use the model you find to help you understand the geometry of the situation.

In this booklet, instead of a ladder, you'll lean a chair against a wall, and measure the distance from the wall to the bottom of the chair, and from the floor to the top. Then you'll graph them and try to find the function that relates those two quantities. In a way, this is the reverse of the traditional ladder problem. Instead of going from the formula to a specific number, you'll go from numbers to the formula.

That means that the point is not a particular answer, but rather a relationship.

We hope the formulas and the geometry will make more sense because they're about something real. Real data is messy and confusing. Still, learning to handle real data is important.

In the full book, there are many nonlinear situations, and for those, you should really use technology to help with the graphing. In this excerpt, where everything is linear, we've tried to set it up so students can plot "best guess" lines by hand.

What will be hard in this book is not the algebra, or the geometry, or even the data analysis. The hardest part will probably be connecting up the *situations* to all the math. This will often require common sense and learning to think mathematically about the situations.

Book Pages

The activities are numbered. Each activity has a student page followed by one or more teacher pages. Teacher pages include "answers," or at least a sample graph. Some activities also have templates, or other pages suitable for copying.

You could just project the student page, and have students work in notebooks if you wish.

Materials

In this excerpt, you will always need rulers and grid paper. Some activities need protractors. Activities about circles may need a variety of round objects, e.g., jar lids, bicycle wheels, etc. Other activities need simple objects such as books or cups.

Prediction

The student pages of this book often ask them to predict. As of March 2014, we usually ask something like, "what will the relationship look like?" In this (linear) booklet, that means,

> *Before you take any measurements, think about the situation and sketch the graph you think you will get when you actually plot measurements*.

This turns out to be a difficult step for students and teachers alike. One strategy is to collect the predictions the day before you spend class time on the measurement.

These predictions serve several purposes:

- They help students think about the situation beforehand.
- They give students a chance to be surprised.
- They give you, the teacher, a chance to make sure students understand what's being asked.

Student predictions often start out being terrible, but they improve with practice. Help students say what makes a good prediction. This can start with something as simple as having labeled axes or realistic values. As students get more experienced, they get better at following the graphing rules, estimating distances, and inferring the shape of relationships. Once they get there, they can even start predicting equations for their models.

You can help make prediction meaningful and effective.

- At the end, always ask students to compare their predictions to reality. If you do reflective writing, this is a good topic.
- Be sure to ask students what was *right* about their predictions. Only then ask how they could have made better ones. Finally, ask about surprises: where did the graph turn out way differently than they thought?

Principles

Working with data is emerging as an important skill. There are important "habits of mind" to adopt that may never have been part of your math curriculum. Here is a partial list:

- Limiting Cases. It often helps to measure and reason about special cases, especially ones at the edges of possibility. The model you create must work properly at these special cases—and it's easy to check that out.

  So if you're doing that Pythagorean ladder problem, your formula had better work when the ladder is flat up against the wall, and when the ladder is flat on the ground.
- Residuals. When you create a mathematical model to fit data, look at the residuals. If the model is good, there will be no pattern in the residuals—they will look random—and they will be centered around zero.
- Enter only the data. Whenever possible, enter actual measurements, and let technology do all the calculations.

Contents

1. Stack of Books

What to Do

In this activity, you will make stacks of different numbers of books. How does the **height** of the stack depend on the **number** of books?

- Predict: What do you think the relationship will look like? (Sketch the graph. You have no data yet, so you can't actually graph it—but what do you think the graph will look like?)

What to Do

Make a stack of books. Measure the height of the stack (as in the illustration) and count the books (in this case, 15). Record the **number** and the **height** for at least five different stacks.

- Record measurements of **height** and **number**, for stacks with different numbers of books.

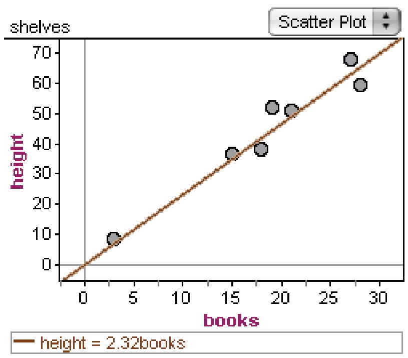
number	height

- Plot **height** against **number**.
- Find a line that fits the points. Find its equation. Be sure you can explain the meaning of the slope and the intercept.
- Be sure you can explain why it makes sense that a *line* fits the points.

Stack of Books

This is a linear situation, in fact, a direct proportion. It's suitable for students with little experience in this sort of thing, or as a quick introduction to this type of activity for more experienced students.

Here is sample graph with a good line superimposed. **Height** is in centimeters.



These students got new books for every stack, and they were not all the same size. Notice that they did not find a **height** for every number of **books**. That's OK.

As you might expect, the slope is the (average) thickness of a single book, and the intercept should be zero. Do not be surprised if students who have already studied slope and intercept do not immediately understand this. Students may have had little experience applying what they know from algebra in a situation with measurement. There are several confusing aspects:

- The variables are not named x and y.
- The points will not lie perfectly on the line.
- The numbers will be messy decimals.

You can help students using several strategies:

- Have them substitute x and y briefly for the real variable names so they can figure out what corresponds to what in their "$y = mx + b$" pattern.
- Work to help them find the meaning of m, the slope. They will say things like "it's rise over run." Push them to tell you what that means in the context of the problem. Once they say, "if you increase the number of books by one, the height will increase by 2.15 cm," you're getting close but not there—you really want them to interpret that true statement in context, namely, "each book is 2.15 cm thick."
- For the intercept, again have them think about the situation: "If there were zero books in your stack, how tall would it be?" If it helps, have them enter that point. In Fathom, it is appropriate to set **Lock Intercept at Zero** in the graph's context menu.

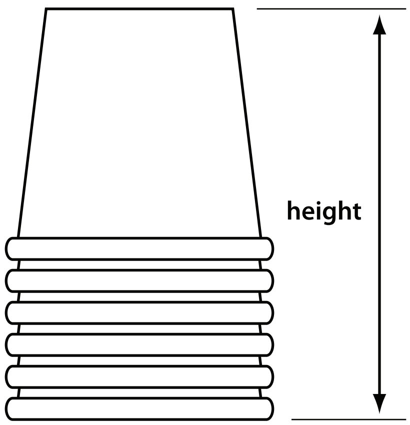
Contexts And Materials

You don't have to use books, of course. It might be good for students to have several different experiences with analogous situations. For beginning students, however, it is important that the books (or whatever) be as close to the same size as possible, e.g., copies of the same book.

Other ideas: stacks of blocks or coins; beans in a graduated cylinder vs. volume; pages of a thick book (note intercept!); distance traveled vs. number of steps; distance vs. number of tiles on a wall or floor; etc.

2. Stack of Cups

In this activity, you will make stacks of different numbers of cups. The cups “nest” inside each other.



number	height

How will **height** be related to **number**?

- Predict: What do you think the relationship will look like?

What to Do

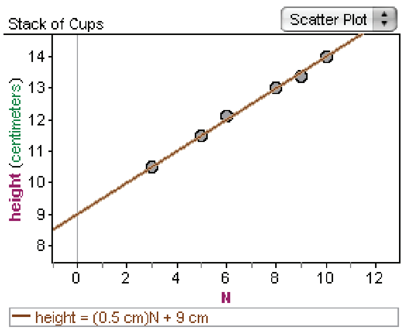
Make a stack of cups, all nested together. Measure the height of the stack (as in the illustration) and count the cups (in this case, six). Record the **number** and the **height** for at least five different stacks.

- Record measurements of **height** and **number**, for stacks with different numbers of cups.
- Plot **height** against **number**.
- Find a line that fits the points. Find its equation. Be sure you can explain the meaning of the slope and the intercept.
- Be sure you can explain why it makes sense that a line fits the points.

Stack of Cups

We put this early in the book because it's linear, so it takes less mathematical baggage to approach. You don't need to have mastered algebra to understand this context and figure out the formula.

But it's not perfectly simple either. Even though you can use the geometry of the situation to understand the data, the meanings of the slope and intercept are subtle. Here is a sample graph with a good line superimposed.

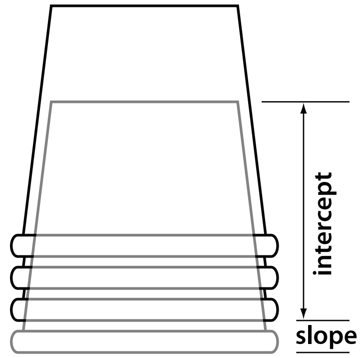


A traditional "meaning of slope" is "the height you add to a stack with each additional cup." This is correct but disconnected from the context. The intercept, however, is worse: "the height of a stack of zero cups." How could that be?

A key question is, based on your formula, *how tall is one cup*? Curiously, it's the slope *plus* the intercept. There are at least two ways to see why this is true:

- If you plug **number = 1** into your formula, you get slope plus intercept. This is a purely algebraic approach—efficient, but not showing understanding of the situation.
- If you make a diagram showing the size of slope and intercept, you can see that the size of a whole cup is their sum. This shows a good connection to the context, but is inefficient (or fragile) because "seeing" the intercept is so strange and hard.

The upshot is that we want students to have both understandings and be able to relate them.



What is really happening here is that the situation (unlike stacking without nesting) really doesn't fit cleanly with the slope-intercept form. Whereas we would ordinarily plug in zero for **number** to test our predictions—it is usually good to test limiting cases—this time zero gives us a bogus answer. Does that mean algebra is bad? No. But it does mean you can't use it without thinking.

<Point-slope alternative?><Limits of domain>

Other Questions to Ask

- What problems did you face, measuring the stacks?

Context Note

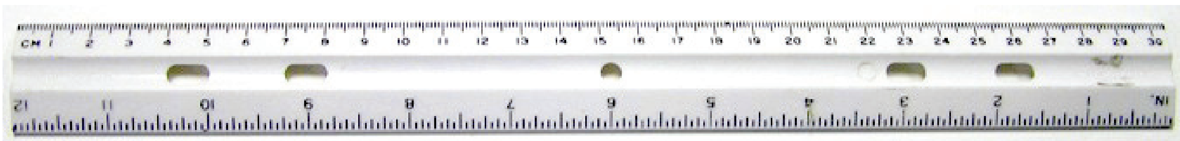
This classic situation appears frequently in math problems. Another context you will see is nested shopping carts: how does the length of a train of shopping carts depend on the number of carts in he train? Data on actual nested shopping carts appears in the eeps Data Zoo. (**http://www.eeps.com/zoo/index.html**)

3. Opposite Sides of the Ruler

In this activity, you will relate the numbers on the opposite sides of your ruler to one another.

What to Do

Take a traditional U.S. student ruler—the kind with inches on one side and centimeters on the other. For at least seven spots on the ruler, record what numbers are opposite each other.



How will **inches** be related to **centimeters**?

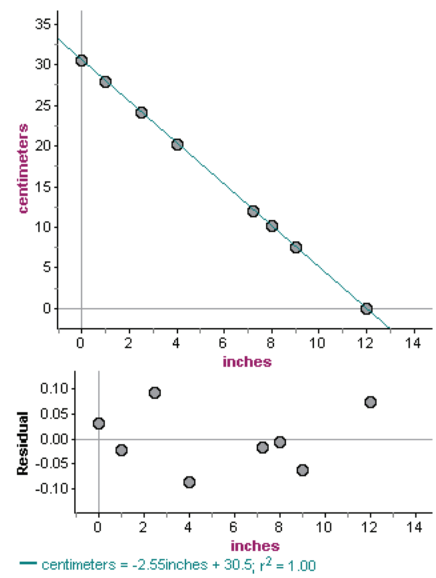
- Predict: What do you think the relationship will look like?
- Record at least seven measurements of **centimeters** and **inches**. For each measurement, pick a spot on the ruler and record the numbers from the two sides of the ruler.

inches	centimeters

- Plot **centimeters** (on the vertical axis) against **inches**.
- Find and explain a mathematical function that fits the points. Be sure you can explain the meaning of any parameters.
- Be sure you can explain why the form of your function makes sense.

For most rulers of this type, the inch and centimeter scales run in opposite directions. This means that although the relationship will be linear, larger numbers on one scale will match up with smaller numbers on the other, giving a negative slope.

Here is some typical data. We have plotted a least-squares line and constructed a residual plot.



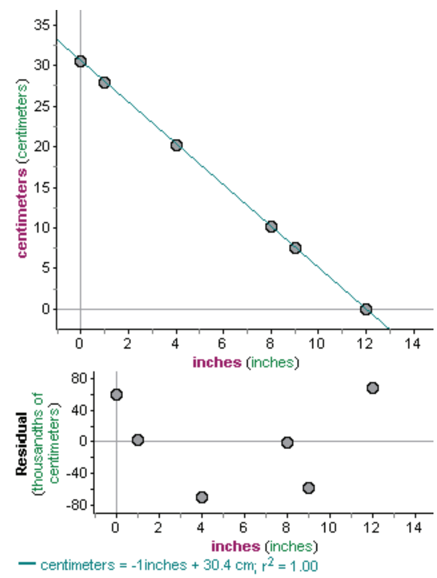
Here are some important questions for discussion:

- What goes on the *y* axis?
- What are the limiting cases? Why are they important?
- If you find a number outside the regular domain, such as a point with **inches = 15**, what could it mean?

The Units Issue

When we try this activity in class, some students get a slope of about −2.54, some get a slope of −1.0, and some get a slope of −0.393. What's happening? This turns out to be in interestingly deep question.

In the illustration at left, the slope value means that there are 2.55 (close to the correct value, 2.54) centimeters in one inch. But look at the next illustration:



It's the same data (minus a few points) but now the slope is −1. The data even have the same values.

How can the data have the same numbers and the computer gives you a different slope?

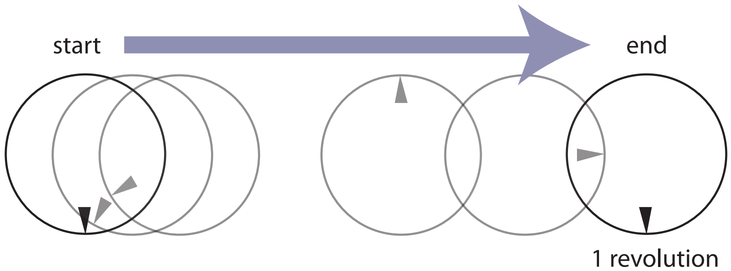
It's because in the second example, the student entered the data with units. Let's compute the slope in the first example between the two endpoints, which are (0, 30.5) and (12,0): we get (0 − 30.5)/(12 − 0), or 2.54. But in the second example, the same calculation has units, so we get

$$m = \frac{(0\text{cm} - 30.5\text{cm})}{(12\text{in} - 0\text{in})} = -\left(\frac{2.54\text{cm}}{1\text{in}}\right)$$

But 2.54 cm is the same as 1 inch, so the value of the fraction is 1.0, and the slope is −1. The first example relates the *numbers*, but the second relates the *distances*. Let that sink in!

4. Rolling Rolling Rolling

In this activity, you will roll a round object on the tabletop, and compare the number of revolutions to the total distance traveled.



revolutions	distance

What to Do

How will **distance** be related to **revolutions**?

- Predict: What do you think the relationship will look like?
- Record measurements of **distance** and **revolutions**.

  Take your round object (whatever you instructor tells you to use) and set it up so the "wheel" is ready to roll on the tabletop. Mark your starting location. Then roll it carefully exactly one revolution and record how far it went. (See the illustration.)

  Use different numbers of revolutions in subsequent measurements (but always start over from zero).

  Note: your numbers of revolutions may not all be whole numbers.
- Plot **distance** against **revolutions**.
- Find and explain a mathematical function that fits the points. Be sure you can explain the meaning of any parameter.
- Be sure you can explain why the form of your function makes sense.

Rolling Rolling Rolling

Materials

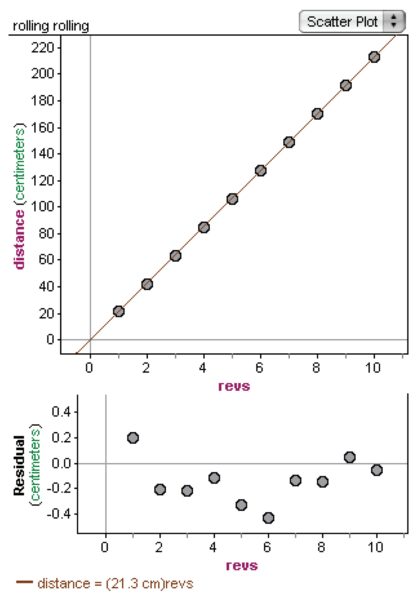
You need round things and rulers or tape measures.

Coins can be hard to roll without slipping. Jar lids are much easier, though they need to be marked. If you can mark the wheels, toy cars work, too. Whole cans or jars are easier to roll than jar lids.

For a larger-scale experience, a bicycle is terrific. Set it up with the valve stem straight *down* and roll the bike forward until it's down again. Then you know you have gone one revolution. Measuring distance can be a challenge because it is so far, but be creative: you could use nonstandard measures such as floor tiles or sidewalk cracks or yards on the football field.

Results

Here is a graph with a residual plot for a 14.5-ounce can of Trader Joe's diced tomatoes:



Discussion Questions

- What variable goes on which axis? Why?
- Why don't the points line up exactly?
- What's the easiest way to measure circumference?

Circumference Intuition

Most of us underestimate how long a circumference is. If you ask students to predict where the bike (or jar lid or whatever) will be after one revolution, the prediction will generally be too short. So have students actually make that prediction: it will help them develop a better intuition about circumference.

Meaningful Parameters

The relationship should a direct proportion, but what is the parameter? Here you have some choices. If you simply use

distance = C * revolutions

then **C** is the circumference of the wheel. It's easy to get the value of **C** in Fathom using a movable line (pegged with **Lock Intercept at Zero**). In our illustration, it's 21.3 centimeters. But what does that number mean?

Alternatively, suppose you make a slider named **R**, and plot

distance = 2 * pi * R * revolutions

Then your parameter **R** is the radius.

Notice how we put the additional constants **2** and **pi** (Fathom and Desmos both know the value of pi) into the model, into the function. If you just have a calculator do a least-squares fit, you'll get **C** (as before). But if you're actually trying to determine the radius, you could mess up trying to decide whether to multiply by 2π or divide. Putting the constants in the model prevents this.

Another alternative is to make two sliders: make one called **D** and set it to the diameter of the wheel. Then make another called **P** and use this model:

distance = D * P * revolutions.

Vary **P** until the line fits the data, and you have found an estimate for π.

5. Circumference

In this activity, you will explore the circumferences and diameters of round objects.

What to Do

For as many different round objects as you can, measure the circumference and diameter. Try to get a wide range of sizes.

How will **circumference** be related to **diameter**?

- Predict: What do you think the relationship will look like? If you can, be precise and quantitative.
- Record measurements of **circumference** and **diameter** for as
- Plot **circumference** against **diameter**.
- Find and explain a mathematical function that fits the points. Be sure you can explain the meaning of any parameter.
- Be sure you can explain why the form of your function makes sense.

Circumference

As you might expect, the relationship is a direct proportion, and π is the constant of proportionality.

This activity is closely related to *Rolling Rolling Rolling* (page 16), but it's not the same. There the proportion arose because twice as many revolutions of the same circle got you twice as far. Here we look at different circles and don't revolve them.

Materials and Procedures

It can be hard to get accurate measurements for circumference and diameter.

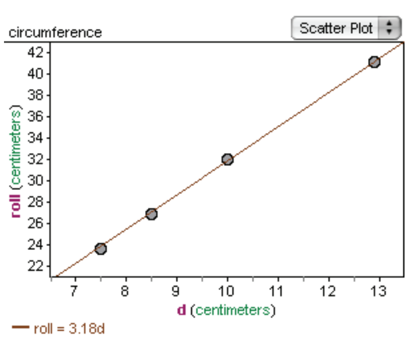
Less-experienced students may need to be told that if you can't figure out where the center is, the diameter is the longest distance across the circle.

As to circumference, three strategies:

- Roll the object once and measure the distance it moved. (Or roll it 5 revolutions and divide.)
- Use a flexible tape measure.
- Use string (string that doesn't stretch!) and then measure the length of the string.

You will need to provide measuring tools and, unless you want students to find their own, a variety of round objects is varying sizes. Jar lids, skillets, tops of tubs, tennis-ball cans, round tables, and bicycle wheels are all candidates. Spheres (such as basketballs), while tempting, cause problems.

Sample Results



This group used rolling to find the circumference. Note the slope: 3.18. Notice also that we do *not* see the origin in this plot.

Discussions

- How did you measure circumference? (The group in the illustration rolled five revolutions, then had the computer divide by 5.)
- How did you measure radius?
- What goes on which axis?
- What range of diameters did you use? Why does range matter, if at all?
- If you used string, how do you know the string didn't stretch?
- If you rolled objects, did they roll straight? If not, why not? How did you adjust?
- How do you know you rolled without slipping? (What happens if you roll back to the beginning after a measurement? Often, if it slips, you don't get to the starting place.)

Extensions

Some people use 22/7 as an estimate for π. How do we know it's not exactly correct? One test is to see how well it predicts the circumference of real objects.

Ask students, therefore, to see if π = 22/7 is consistent with their data. It probably is.

Then, since they (and their calculators, and computer programs) know better estimates for π, have them figure out how big the difference is between circumferences calculated with π and with 22/7, and discuss whether, and in what situations, it is important to use the better estimate for making a calculation.

6. Triangle Ladder

In this activity, you'll study the lengths of the "rungs" of a "triangle ladder."

What to Do

Make a triangle ladder!

- Sketch a triangle, a pretty big one, an a sheet of paper. Don't use a ruler, but make it fairly carefully. Label the vertices *P*, *Q*, and *R*. It doesn't have to be a special triangle (e.g., isosceles).
- Make at least five segments parallel to $\overline{PQ}$ that extend from $\overline{PR}$ to $\overline{QR}$. These are the "rungs."
- Measure and mark down the lengths of all the rungs.

The length of that **rung** will depend somehow on a measurement along the **side** of the triangle (that is, along $\overline{PR}$ or $\overline{QR}$).

- Decide what to use for your **side** distance. Write down, briefly but clearly, how to measure **side**.

How will **rung** be related to **side**?

- Predict: What do you think the relationship will look like?
- Record measurements of **rung** and **side**, and plot **rung** against **side**.
- Find and explain a mathematical function that fits the points. Be sure you can explain the meaning of any parameter.
- Explain why the *form* of your function makes sense. If you've studied geometry, you should be able to explain it using geometrical vocabulary.
- Fine someone who had a different definition of **side**, and compare your two functions. Figure out how they're related to each other.

Explore: how much did it matter that you drew the diagram freehand, without a ruler?

Triangle Ladder

In theory, this is easy for any student who has studied similar triangles, but in practice it can be confusing. That's why it's so important to look at this utterly typical geometrical situation. Using data might give some students the perspective they need.

The problem is that it's easiest to measure the distances between the ladder "rungs"—and those don't have any particular relationship to the lengths of the rungs themselves.

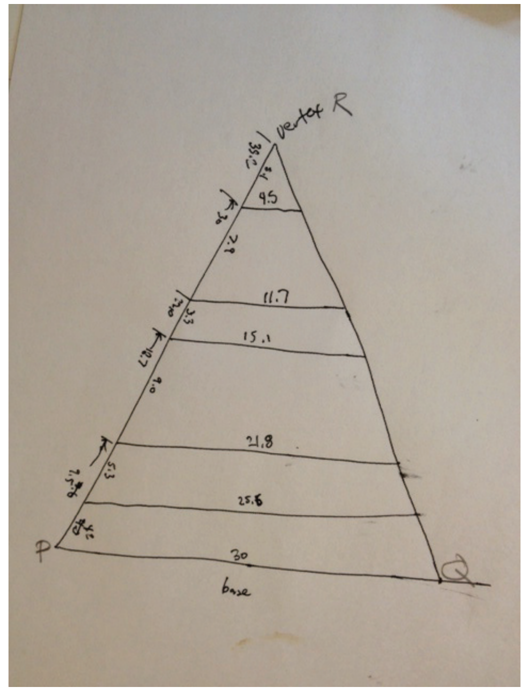
Instead, students have to see that the relevant distances are the total, cumulative distances, either from the vertex or from the base. If they're from the vertex, they get a direct proportion. If students measure from the base, they get a linear relationship with an intercept (and the intercept is the length of the biggest base).

Discussion Topics

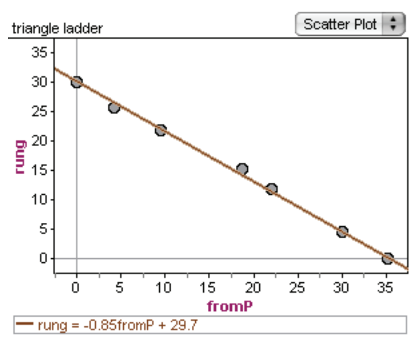
- How did you figure out what to measure for **side**?
- Why do some people have positive slopes and some have negative?
- What do the slopes mean? How could you figure out the slope simply by measuring the big triangle?
- If you made both graphs (from the vertex, and from the base), how would they be related?

Results

Here is a picture of such a ladder:



Here is a graph of some sample data. Here the student used **fromP** instead of **side**, which is fine. Since they are measuring form the base instead of from the vertex, they get an intercept and a negative slope.



7. Isosceles Angles

In this activity, you'll study the angle in an isosceles triangle.

What to Do

Make some isosceles triangles. Your instructor will tell you what tools to use. But if you're doing this on your own, you could just draw the triangles freehand, as accurately as you can "by eye."

You will need a protractor to measure the angles.

- Sketch some isosceles triangles, as large as is practical given your paper. Sketch a variety—some with small (acute) vertex angles, and others with large (obtuse) vertex angles. You need at least five triangles.

- Label each triangle with a number, and label the vertices A, B, and C, where A is the vertex—the point where the two like sides come together. So we'll call the vertex of triangle 4 A_4, and so forth.

- Measure all the angles. Write their values in the angles, and record them in a table. The column headings should be **number**, **A**, **B**, and **C**. (The number of the triangle is **number**.)

How will **A**, **B**, and **C** be related?

- Predict: What do you think the relationship will look like? Since you have three variables instead of two, give some thought to the best ways to express your prediction. It may be that you have more than one graph.

- Find and explain mathematical functions that fit the points. Be sure you can explain the meanings of any parameters. For example, if your equations have any coefficients, why do they have to have the values they do?

- Test your predictions: how good were they? How well do the data support what you claimed?

Explore: if you drew your figures freehand, without a ruler, what difference would it have made if you had been able to make your triangles perfect?

Isosceles Angles

Many students probably know that $A + B + C = 180°$, and that for isosceles triangles with A as the vertex, $B = C$. For those students, this is a perfect opportunity to make really good predictions.

The extra, open-ended challenge in this task is to deal with three variables. If you make a graph, what do you put on the axes? One good strategy is to make *two* graphs: B against C and A against B or C.

Some students may try to make a single graph that relates all three variables. This is an interesting challenge; if a students or a group seems to be getting bogged down, however, consider suggesting that they do something less ambitions (i.e., use two "normal" scatterplots). See if they want to pursue the 3-variable graph elsewhere—maybe as extra credit.

Some students may give up on graphs entirely and rely instead on formulas for their predictions. Their challenge will be to evaluate their predictions without a graph: how can they tell of their guess even has the right shape? It's not impossible; for example, they might predict that the sum of the angles is 180°, and test the idea by adding the three angles and comparing. This is fine, but in the debriefing, ask the class what you get out of a graph (or a set of graphs) that you don't get out of calculation.

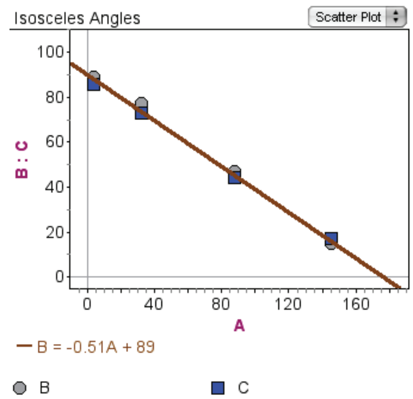
Measuring Angles

Measuring angles well seems to be a challenge even for otherwise accomplished students. If your classroom is like many, you have a motley collection of protractors, and students are inexperienced. This activity gives them some needed practice.

The biggest danger is angles slightly larger than 90°; if students record a 95° angle as 85°, it can mess up their data. (This can be a good thing if students see that there is something wrong with the point that doesn't fit in the graph; then they can re-measure and fix the problem.)

Results

In this example, the group has put **A** on the horizontal axis and plotted *both* **B** and **C**. They have not, however, looked at limiting cases.



Notice two things about the graphs we get:

First, the formula for the line is much more understandable with A on the vertical axis: it follows directly and easily from $A + B + C = 180°$. The other one is just as true, though.

The graphs do not immediately "look" like the relationship—at least not the way we might usually think of it.

Simplifying the Task

If all this open-endedness is too much for less-experienced students, you could simplify the task in two ways:

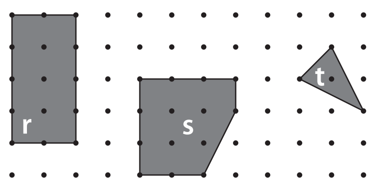
- Have students ignore angle A and study the relationship between B and C, that is, have them "discover" that the base angles of isosceles triangles are equal; or
- have students relate A to either B or C. Then they will find that $A = 180 - 2B$ (or some equivalent expression).

You could also make students' lives easier (but a little less fulfilling) by giving them a template to start with. See the "Vertex Angles Diagram" on page 25.

8. Pick's Theorem

In elementary school, you may have played with cool manipulatives called geoboards. There was probably a grid of plastic poles, and you would stretch rubber bands around them to make shapes. These shapes are polygons, by the way: they have straight edges.

The key concept behind many geoboard activities is area. In order to find the area of some odd shape, you would not use formulas, but rather ingenuity. Look at the three shapes below:



The area of shape **r**, a rectangle, is 8.

What about **s**? Think of it as a square with a corner cut out. The square has an area of 9. The corner is a triangle; if you took two of those (and rotated one a half-turn) you could put them together into a 1-by-2 rectangle. That means the triangle has an area of 1, so shape **s** is (9 – 1) or 8 square units.

Shape **t** is like **s**, but harder to see. It's a 2-by-2 square (area 4) but with three triangles cut out—two "1"s and the small one, with area ½. So the total cut out is 2 ½, which leaves 1 ½ in the triangle.

Doing the Pick

This gets harder as you get stranger and stranger shapes. Georg Alexander Pick found an elegant way to figure out these areas, and you're about to find it as well. Here is what you do:

Use the grid on the handout to make a bunch of polygons. All of their vertices must be on the dots.

- For each polygon, find the area **A**.
- Also, count the number of boundary points. That's **B**. For polygon **s**, **B** = 10.
- Then count the number of interior (contained) points. That's **C**. For polygon **s**, **C** = 4.
- Assemble all this data in a table.
- Use your data to figure out how to calculate **A** using **B** and **C**.

This means you're going to make a scatter plot with **A** on the vertical axis. But what do you put on the horizontal?

Controlling Variables

One good strategy is to *control variables*. That means looking at all of your predictor variables (a.k.a. independent variables)—in this case, **B** and **C**—and holding one constant while you vary the other.

For example, you might look at only your polygons with **B** = 3, like triangle **t**. (These will all be triangles, but of different shapes and sizes.) Then you could see how the area **A** depends on the number of interior points **C**.

Then hold **C** constant: make shapes with the same number of interior points but with different numbers of boundary points (**B**). Then see how **A** depends on **B**.

Gradually find a formula for **A** that has both **B** and **C** in it, and works for all cases.

Using Fathom to Help

Fathom can help you with this. If you have a bunch of data for all different values of **B** and **C**, but you want to see how **A** depends on **C**, do this:

- Make a scatter plot with **A** on the vertical axis and **C** on the horizontal.
- Drag **B** into the *middle* of the plot, holding down the **shift** key before you drop it.
- The plot will now indicate the different values of **B** in different symbols.

Pick's Theorem

Pick's Theorem

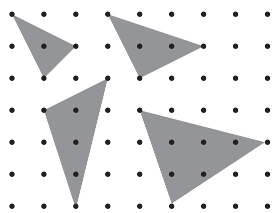
This is for mathematically more mature students.

As written, the activity gives students very little scaffolding. This is intentional. It would take a lot of paper to write a comprehensive set of instructions for how to control variables—and who would read it? This is the sort of thing you have to experience, and wrestle with, to understand.

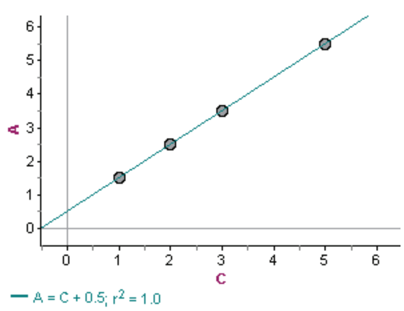
Then the idea of putting the general formula together is probably different from the sort of thing that students have done before.

Nevertheless, using data and graphing, we think that this experience is more accessible to more students than before.

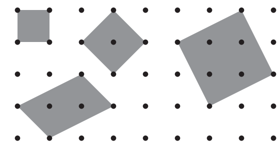
Here, for example, are 4 triangles with **B** = 3. The areas are 1 ½, 2 ½, 3 ½, and 5 ½, with 1, 2, 3, and 5 interior points. I smell a pattern...



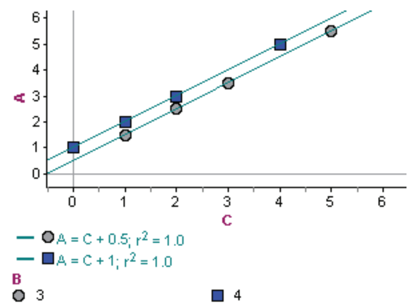
If we plot **A** against **C**, the number of interior points, and add a least-squares line[1], we see this:



Having gotten this far, you can do as the student page suggests and control the number of interior points, or you could control for boundary points again, but with 4 points instead of 3:



If you use the Fathom strategy on the student sheet, this yields:



Note that we can already predict. For example, it looks from the graph as if a polygon with **B** = 3 (marked by a circle) and **C** = 0 should have an area **A** of ½. and that's exactly right.

Oh: you want the answer? We won't spoil it for you here, but you can find it easily on the web. They often use **E** for edge or **I** for interior.

Note that this is different from other EGADs activities because there is no measurement error (you would never get an area of 11.94, or 3.2 points inside the polygon). Therefore all points should fit the model exactly.

1 We could use a movable line instead, but in this case, a least-squares line saves us some effort.

9. Cornbread Are Square

"Pie are round; cornbread are square"—Anonymous

In this activity, you will explore the areas of circles. This might be a good first quadratic activity.

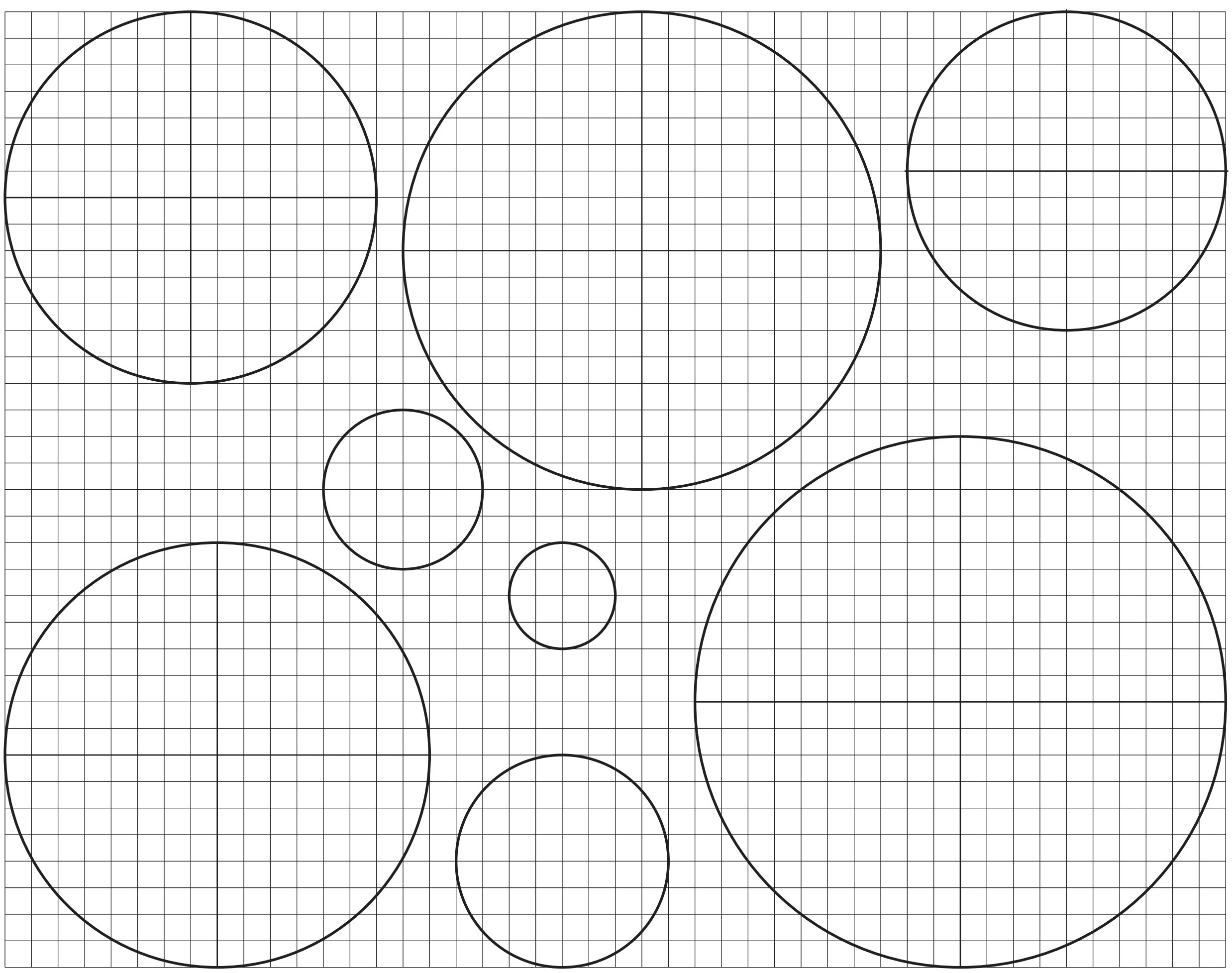
What to Do

Count the squares in each of the circles below (or you could make your own using graph paper). Count the area in the partial squares as well, estimating how many whole squares they make up together.

You will measure area and radius in the units of the grid.

How will **area** be related to **radius**?

- Predict: What do you think the relationship will look like? If you can, be precise and quantitative.
- Record measurements of **area** and **radius** for as many circles as you can.
- Plot **area** against **radius**.
- Find and explain a mathematical function that fits the points. Be sure you can explain the meaning of any parameter.
- Be sure you can explain why the form of your function makes sense.



Cornbread Are Square

Instructor Notes

"Pie are round; cornbread are square"—Anonymous

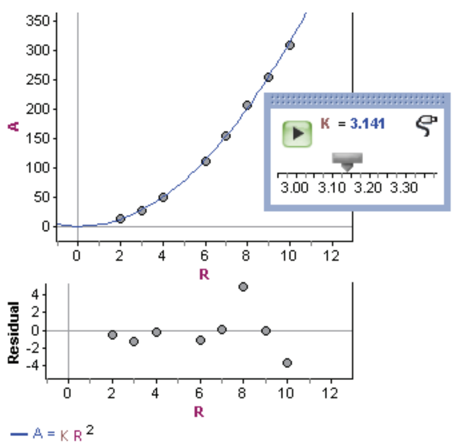
Even if students can recite the formula $A = \pi r^2$ in their sleep, the data approach is interesting, because

- They may not have thought about what area actually means in a while;
- They may never have had this formula confirmed for themselves; and
- Students may not even think of this formula in this context.

We expect students to count the squares to get the area. They could count a quarter of the squares in each circle, then multiply by 4. That's fine.

The most common way to count the partial squares is to estimate which partials go together to make up whole squares, and shade them in as you count.

Here is some pretty good data. The residual plot is helpful and illuminating; students will see that the larger circles have more leverage in the residual plot.



Why Use Functions?

You could just calculate "pi" for each circle, taking the area and dividing by the square of the radius. Why go to the trouble of plotting the function?

For one thing, it shows us the relationship, not just the number.

for another, the numbers get more accurate with larger radii; if we simply averaged the πs, we would get a skewed result.

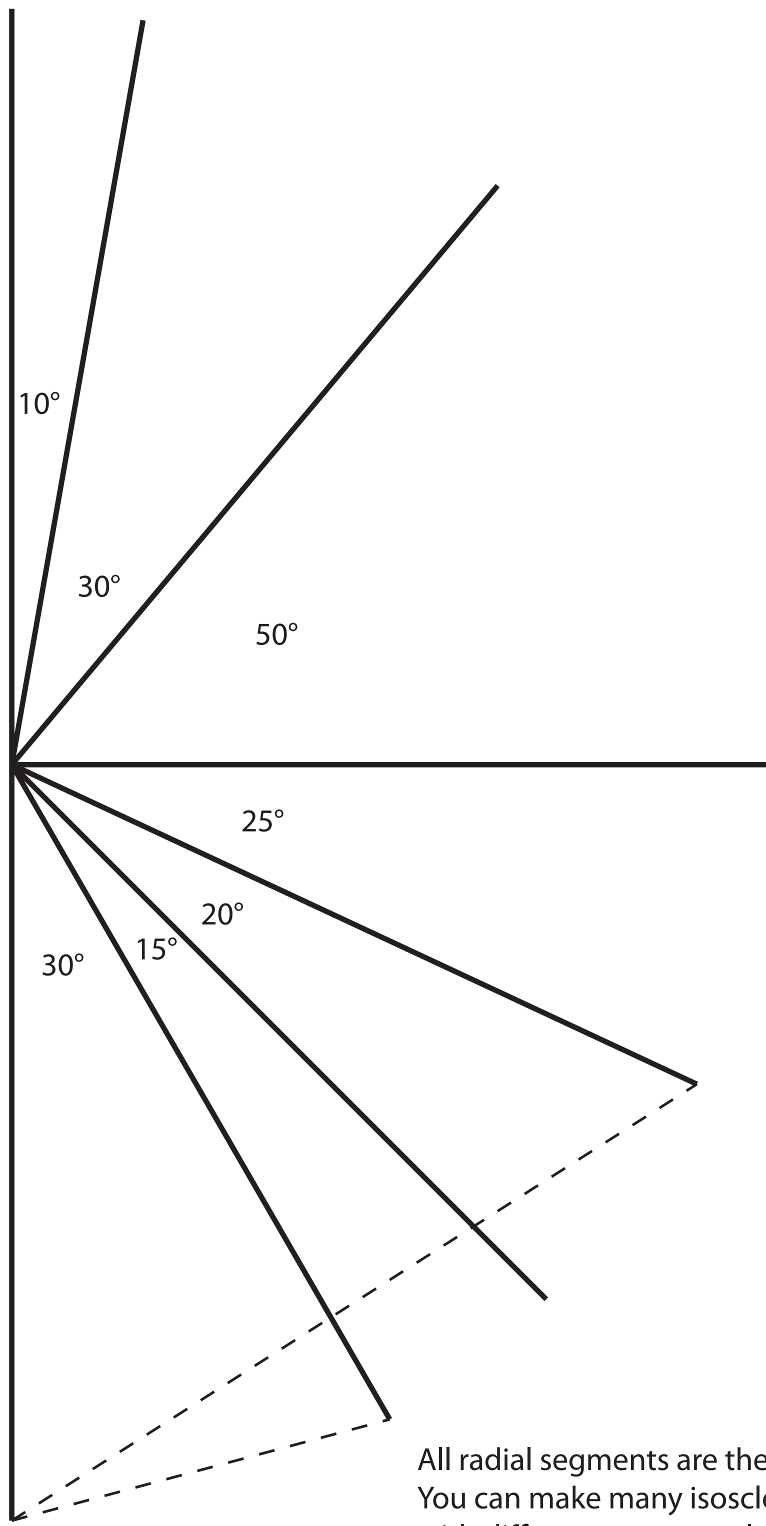
Note: you could plot the area against the square of the radius; then you should get a straight line that (limiting cases) passes through zero, and has a slope about π.

Alternative Approach

Instead of counting up partial squares, you could have students "bracket" π by doing an upper/lower limit dance:

- Count the squares that are *entirely* within the circle. This "area" is the minimum area.
- Next, count the squares that contain any part of the interior of the circle. (That is, the interior squares pus all squares the circle goes through.) This is the maximum area.
- Make one graph of the minima, one of the maxima, and find the parameter that corresponds to each one.
- Notice how the larger the circle, the closer the minimum parameters is to the maximum.

Vertex Angles Diagram



All radial segments are the same length.
You can make many isoscles triangles
with different vertex angles.
The two dashed lines are examples
of bases for 30° and 65°.