a game about resources and cooperation

This is a game you play first and talk about later. Keeping with that notion, we'll describe the rules first and worry about the rationale and the underlying structure at the end of this handout.

#### Materials

- a lot of poker chips in four colors. See the setup notes on the next page.
- a "bank" to keep the chips in
- a bag (paper is OK)

### The object

of the game is to have as high a value of chips as possible at the end of the game—at the end of the tenth turn.

### Setup

At the beginning, players sit around a table. A quantity of chips (depending on the number of players; we'll get to that later) is placed on the table. The rest of the chips are kept in the "bank." The bag should be handy.

Explain the values of the chips: gold are worth five, blue are three, red are two, and white are one.

### Each turn

- 1. Each player, in turn, takes two chips from the table for himself or herself—chips of any value.
- 2. Each player pays "subsistence" of five to the bank—any combination is OK as long as the *value* is five.
- 3. Put all remaining chips from the table into the bag.
- 4. Each player draws two chips blindly from the bag.
- 5. Each player "doubles" those two chips from the *bank*; the four chips (the two drawn and their doubles) go onto the table.
- 6. All the chips left in the bag return to the table.

On the next turn, a different player goes first.

# Special Very Important Rule

If anyone fails to pay subsistence in any turn, *everybody loses*.

## Setup and Materials Notes

How many players? Up to about six per game. You can run many games at a time if you have enough chips and bags.

How many chips? A good rule of thumb is two to three chips of each color per player. In the above example, we used two and a half. If you have 30 students in your class, you would need, at a minimum, 2 chips per color times 30 players times 4 colors = 240 chips minimum. You will actually need up to twice that many if you include the bank. 600 chips is a comfortable amount.

Chips often come in three colors, with twice as many whites as red and blue; you can mark half the whites to make the gold chips. In 2014, a set of 100 cheap plastic chips (50 white, 25 red, 25 blue) typically sells for \$5–\$10.

You can also save on chips if each "player" is actually a group that decides together on their moves. This policy makes the game even *more* a lesson in cooperation as well as saving on equipment.

#### **Common Questions**

Can we trade chips? Yes.

Can we make change in the bank when we pay subsistence? Yes. For example, if you have two blues, they're worth six. Turn them both in and you get a white in change.

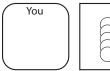
If somebody refuses to pay subsistence, but I want to keep playing, can I pay their subsistence? Yes.

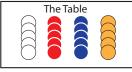
Are there simpler rules? Sure. Use one chip per color per player and omit the bag. After each player has taken two chips and paid subsistence, simply double the chips on the table to prepare for the next turn. This is much simpler, but much less interesting for experienced students: no probability, and the "lesson" is really out in the open.

# An Example

The hard part is the business with the bag. An example may help.

Suppose there are two people playing, you and Ned. There are twenty chips on the table, five of each color. Here's the situation at the beginning:

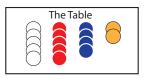






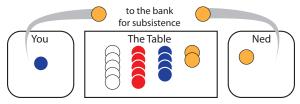
Step 1: You each take two chips. Ned takes two golds (total value: ten). You take a blue and a gold (a total value of eight).







Step 2: Paying subsistence. You each pay one gold to the *bank*. (Not to the table or to the bag.)

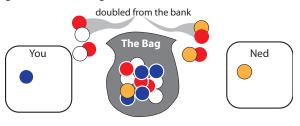


In step 3, the remaining 16 chips go from the table into the bag. Shake up the bag.

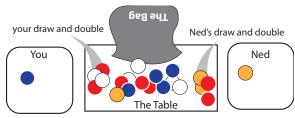
In step 4, you pull two chips out blindly and so does Ned. You get a red and a white. Ned pulls out a gold and a red. (Now the bag has twelve chips in it.)



The instruction for step 5 says to double them, so you get a red and a white from the bank. Ned gets a red and a gold.



In step 6, those eight chips—two golds, two whites, and four reds—go to the table. So do the twelve still in the bag, for a total of twenty.



You still have your extra blue chip and Ned has his gold. They're your own.

This may seem a little complicated at first, but it's pretty easy once you get the hang of it. There are three essential parts to each turn: taking chips, paying subsistence, and "replenishment," in which you dip into the bag to find out what will replace what you took.

Play ten turns. See if you can do that without someone becoming unable to pay their subsistence. Remember, if that happens, everybody loses.

## What is this game about?

On the surface, it's about probability. People pull things out of a bag and see what happens. Not all probability is rolling dice.

It also creates a context for talking about probability. What do you think will happen when we dip for chips? Why? It matters because you care what chips will be available for you on the table.

Even more important, though, are the consequences of cooperation (or un-cooperation), and how they are attached to the probability, or more broadly, to the *mathematical model* the game sets up. After all, what's on the table depends on what *other* players take off of it, not just on you.

It should be clear from playing the game—but not from the rules—that what's good for you in the short run (taking gold chips, each worth five) is bad for the whole group in the long run. In fact, if everybody takes two golds the first time through, it will be virtually impossible for the system to recover. Everyone will fail to meet their subsistence payments before the game ends. (Consider: what happens after someone takes the last gold chip?)

So this game is about using renewable resources that are shared in common with others. In fact, this game presents the players with a "Commons" problem, as described by Garret Hardin in a 1969 article in *Science*, "The Tragedy of the Commons."

We have a lot of Commons problems these days. Consider the harvesting of just about anything, and there is a Commons problem underlying it. Water is such a resource throughout the American West. Ask anybody in California's central valley about the water table there. Ocean fisheries are commons as well, with fisherfolk—or nations—as the players. In this game, taking the last gold chip makes gold literally extinct: there are none left to reproduce.

How can we learn to cooperate when it appears to be in our best interest to be selfish and antisocial? In a traditional cooperative activity, kids learn to cooperate partly because the setup makes it deliberately easy. Here the object of the game is individual: get as much as possible as an individual. So the setup is deliberately divisive.

Yet the situation is not all bleak; it's possible for everyone to win a Commons game too. Nature is bountiful, just not infinitely so. It's exploitation that leads to disaster, not judicious use.

And in this game, students can learn how collaboration can lead to genuine, sustainable wealth. To that end, it's great if, after a discussion, you can let students *play the game again*. If students understand the lesson, it takes only a few moves for them to work together to "beat" the game.

#### Other Math Extensions

Keep wall charts and graph the number of whites (and golds) on the table as a function of turn number. Graph the wealth of the wealthiest player or the combined wealth of all. Graph whatever the students think might be relevant!

Discuss what the likely draw will be on each replenishment. (Given ten whites in the bag and five reds. You're drawing two. What are you most likely to draw? This question can be answered at many different levels of sophistication.)

Discuss what physicists would call the continuity principles involved. Think about the table: how do chips leave the table? How do they return? The first big realization is that the number of chips remains constant. Later you get to the harder question of what affects the distribution of chips on the table.

Explore the various numbers in the game, how they are related, and why they are important. Here are some, think of more:

- The number of chips on the table
- The value of the chips on the table
- The number and value of chips in the bag
- The number of players
- The number of white chips in the game
- The total value of all chips taken in a turn
- The average value of one chip on the table
- The expected value of all chips drawn from the bag

Think of these numbers as sources of important algebra (or even calculus!) questions as well as probability and "counting" questions.