Polygon Areas Part 1

Your handout has a big circle with degrees marked around it. You'll make *regular polygons* inside the circle and compute their area.

For example, to make a regular 3-sided polygon (also known as an equilateral triangle)...

1. Divide the circle up into 3 parts and mark the vertices (corners) at the right places. That could be at 0°, 120°, and –120°—though there are other choices that also make the correct triangle.
2. Connect the dots to make the triangle.
3. Then connect the center (the dot) to each of those three points. Now you've split the triangle up into three triangles. They should be identical. If they're not, something is wrong.
4. Measuring the base and the height of one of the triangles, compute its area. Use the ruler on the page for measuring.
5. Since you know the area of one of the triangles, compute the area of the big triangle.
6. Compute the area of the big triangle a different way and compare. Did you get the same result? If not, is it close? How close do you think is close enough?

You'll be doing more polygons, so use this table to record your data:

polygon name	triangle						
number of sides	3	4	5	6	8	10	12
degrees per side	120°						
base							
height							
area of one △							
area of polygon							
perimeter							

Now work to fill in the table. Work with others, check each other's work, and be efficient!

Hint: *you don't have to make the whole polygon to find its area*.

Polygon Areas Part 2

Okay, now suppose we want to make polygons with even more sides!

First, try a 24-sided polygon the same way you did in part 1. Think about what makes getting an accurate area hard.

Then develop a way to find the areas of these polygons *without measuring the base with the ruler*:

number of sides	24	36	60	72	120	180	360
degrees per side							
base							
height							
area of one △							
area of polygon							
perimeter							

Now work to fill in the table. Work with others, check each other's work, and be efficient!

Finally, generalize. Suppose you have a polygon with a large number of sides.
The number of sides is *n*.

How many degrees per side? ___________

What's the (approximate) perimeter of the polygon? __________

What's the length of the base of the triangles? ____________

What's the (approximate) height of the triangles?____________

What's the area of each (skinny) triangle? ___________

What's the total area of the polygon? ____________

Summarize: what do you know now about the area of a *circle*?

Polygon Areas (Teacher Notes)

Part 1 is all about measuring and using the formula $A = (1/2)bh$. Along the way, students

- can learn what an *inscribed* polygon is
- divide the 360° of the circle by various friendly numbers in order to make polygons
- see that you can split any polygon up into triangles if you do it systematically
- organize their information and look for patterns

It opens with students finding the area of an equilateral triangle as an example. Here are some tips for that part and then later when they investigate up to dodecagons:

- Watch how students measure. Be sure they put the ruler's zero at one end. The first triangle's base should be about 17.5 units; its height should be 5.
- Encourage students to estimate to the nearest *tenth* of a unit. It's good practice, and you'll learn about how they cope.
- Ask how they decide how to measure the height. One strategy is to split the base, dividing its length by 2. Another, sneakier one it to "aim" for the number of *degrees* that's halfway.
- Make sure students make the distinction between the area of the triangles and the area of the larger polygon.
- Have the class stop briefly before the table to compare areas for the big triangle and the little triangles, and to discuss alternative ways to measure the large triangle's area. You can do that with the square as well, of course: the area should be the square of the base!
- As they work through the table, make sure they understand the trick: you only need one triangle to find the whole polygon's area.
- When the table is completed (possibly on the board) ask about any patterns. The total areas should be increasing. Why? The height is increasing too. Why?
- The perimeter is increasing too, though some student measurements may belie this. Discuss whether it's actually increasing (it has to be) and how it's possible to get measurements that fluctuate a bit.
- Interestingly, the individual triangle areas should be highest for the square. This is subtle, but you can ask, if the triangles are getting smaller, how can the polygon area keep growing?
- Finally, be sure to ask, *suppose we only knew the perimeter. How could we find the base*?

In part 2, students measuring the 24-gon (15°) should get different answers because measuring the base is so hard. But the height is getting stable; it's been getting closer and closer to 10. Ask why. (10 is the radius of the circle; that's why we're using the paper rulers).

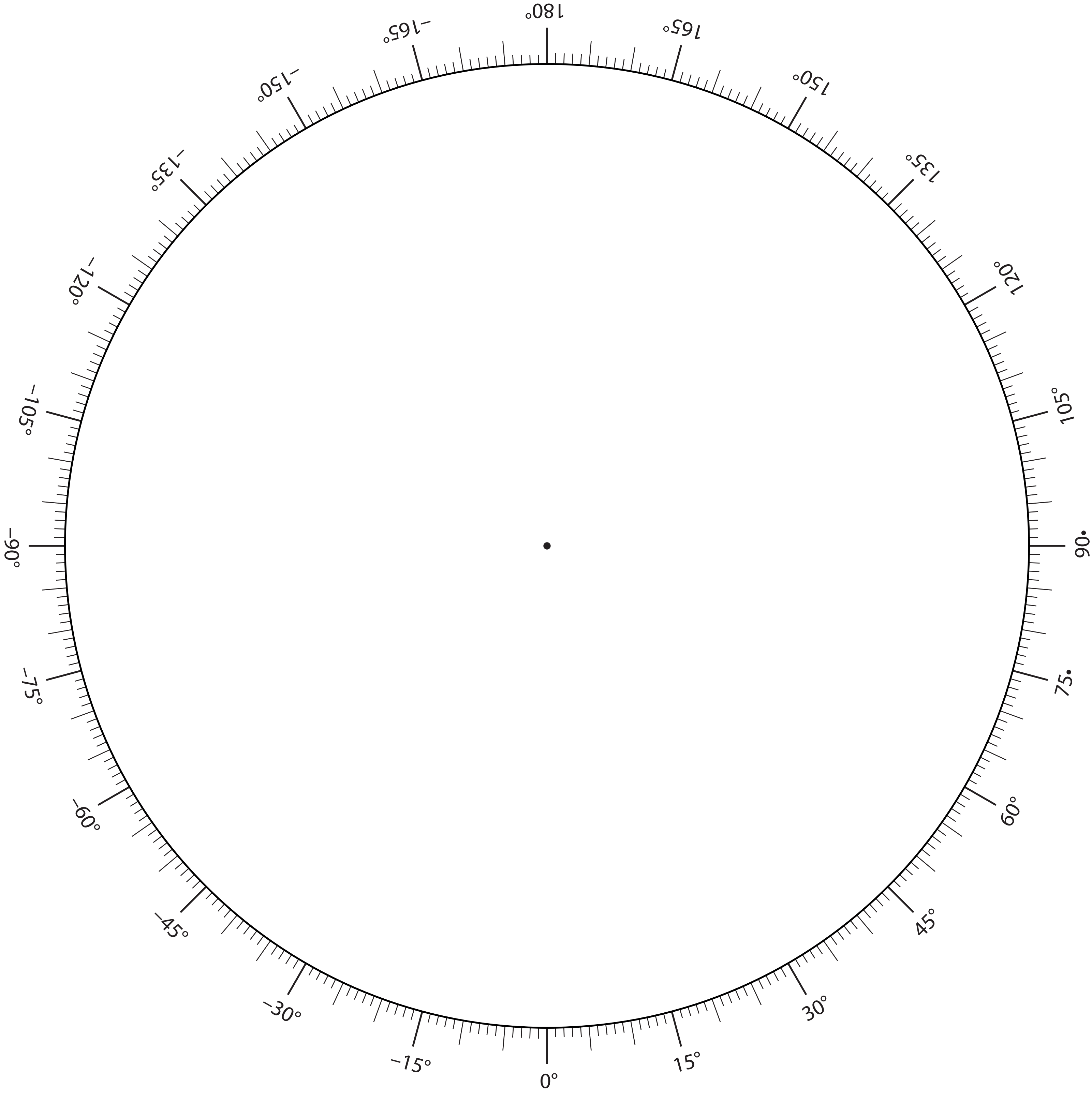
The perimeter has also been getting more stable: it's getting closer and closer to 63—probably fluctuating.

Do not initially tell students to think about the circumference. Let them try for a while to "find the areas of the polygons without measuring the base with the ruler."

If they don't come up with the circumference, ask (about the 24-gon or 36-gon, say), "What do you think the height of this polygon is going to be? (10-ish) What about the perimeter? (63-ish). Given that perimeter, how big is each base? (63 ÷ 24 or 36)" and let them calculate the small areas and the total area.

If someone says, "but you don't have to divide because you're just going to multiply again," you've won. Have *them* explain it.

The generalization step should get you to *A* = 314-ish, no matter what *n* is, because the *n*'s cancel. Then help them see, if they don't already, where the 314 comes from. That is, it's (1/2)(*circumference*)(*radius*) = πr^2 .



cut ruler off and fold