Annals of Plausibility

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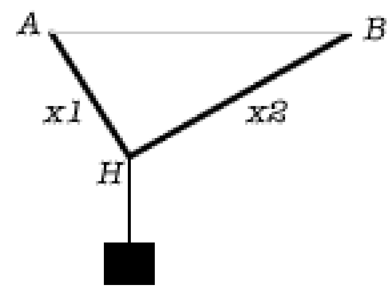
In any case, we would love to hear what you do with these, and would be ecstatic to see Actual Student Work of any quality. Send it along!

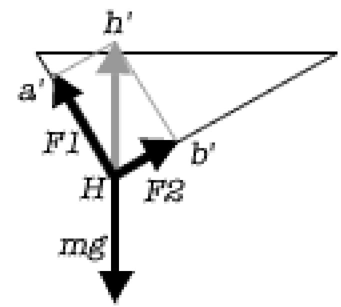
Hanging a Weight from a Horizontal Cord: Balancing the Vectors Using Similarity

P. SNERD AND J. M. W. DOOGIN

ABSTRACT
When a weight hangs from a horizontal cord, the tension in the “short” segment is larger than the tension in the “long” segment. We show that the ratio of these tensions is, in fact, the reciprocal of the ratio of the segment lengths.

Suppose we attach a cord to two points, *A* and *B*, that have the same vertical coordinate. Let us further suppose that the length of the cord is greater than the distance between the two points *AB*. Now we hang an object somewhere along that length. If we neglect the mass of the cord, the cord will form into two straight segments supporting the hanging object.





In the static situation—the object does not move—the forces applied by the two cord segments must exactly balance the weight of the object. These are tension forces, and must be directed along the cord segments, as shown in the figure at right. In that figure, we see the two vectors **F1** and **F2**, whose resultant sum is exactly opposite to gravity’s force on the object, **mg**.

But how long are those vectors? What is the tension in the string? We can easily show that $\Delta AHB \sim \Delta h'a'H$, that is, the small triangles in the lower force diagram are similar to the larger triangle determined by the two cord segments. Therefore,

$$\frac{x1}{x2} = \frac{F2}{F1}. \qquad \text{(EQ 1)}$$

This is in accordance with our experience and with the diagram: to balance the forces, the more vertical tension—which goes along the shorter cord segment—must be larger than the tension on the longer, more horizontal segment. Algebraically, this situation is analogous to that of a teeter-totter: we can rewrite the above as

$$x1 \cdot F1 = x2 \cdot F2 \quad . \qquad \text{(EQ 2)}$$

We imagine that this simple result will make many engineering calculations much easier in the future.

On the Descent of Cotton Balls: a Theoretical Perspective

J. K. FINKLEBOTTOM & P. R. PRIEST

ABSTRACT
Cotton balls fall more slowly than rocks in most situations. We present an extension to the traditional Newtonian view of objects to include free-falling cotton balls.

Cotton balls (which are sometimes made of Rayon) are puffs of fluff, roughly spherical, with a diameter of about 3 cm and a mass between 0.5g and 1.0g. If you drop them, they fall.

It has been observed, however (Galileo and Snerd, 1998) that if you drop a rock and a cotton ball simultaneously from the top of a tower, the rock hits first.

Evidently air resistance slows the cotton ball more than the rock. We suggest that its effect is greater because the cotton ball is lighter.

Our reasoning is this: Each air molecule, on impact, imparts a small force to a falling object. Using the traditional force formula $F = ma$ (Newton, 1687), we see that each collision effectively reduces the gravitational acceleration of any object falling through air by an amount that is inversely proportional to that object's mass (i.e., $a = F/m$). Thus the light cotton ball is slowed more than a comparably-sized (and heavier) rock.

Therefore we should modify the formula for the distance s fallen in time t. Instead of the traditional

$$s = \frac{1}{2}gt^2 , \qquad \text{(EQ 1)}$$

where g is the acceleration of gravity, we suggest that the correct model for falling cotton balls is

$$s = \frac{1}{2}kt^2 \qquad \text{(EQ 2)}$$

where k is an acceleration smaller than g.

Though the truth of our theory seems self-evident, we await confirmation from experiment.

Analysis of Thin-Lens Data

T. FLINTHOFF

ABSTRACT
We analyze image distance data and find an important relationship in optics

A recent investigation has yielded data that relates the distance from an object to a lens (which we will call d_o) to the distance from the lens to its corresponding image (which we will call d_i). A scatter plot of the raw data appears in the margin as Figure 1. One readily discovers that, although the data appear at first glance to be inversely related, no inverse function of the form $d_o = K/d_i$ fits the data.

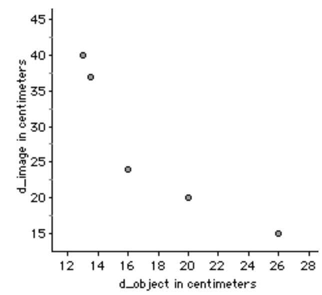


Figure 1: raw lens data

One can, however, transform the data. In particular, we can study the relationship between the reciprocals of the data v_o and v_i, where $v_o = 1/d_o$ and $v_i = 1/d_i$.

We discover that while the relationship between d_o and d_i is complicated, the reciprocals v_o and v_i are linearly related, as shown in Figure 2.

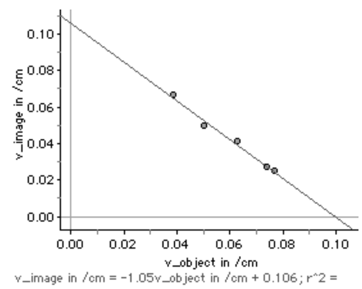


Figure 2: reciprocal lens data

The least-squares best-fit line in Figure 2 has the equation

$$v_i = -1.05 v_o + 0.106 . \qquad \text{(EQ 1)}$$

Rearranging, and re-substituting the original variables, we have

$$\frac{1}{d_i} + \frac{1.05}{d_o} = 0.106 . \qquad \text{(EQ 2)}$$

While we do not understand the full significance of these two constants—1.05 and 0.106—the fact that one is almost exactly 10 times the other can hardly be a coincidence. Apparently, each lens has associated with it a constant we call its “Flinthoff number” φ, and that the universal equation relating image and object distance is

$$\frac{1}{d_i} + \frac{\varphi}{d_o} = \frac{\varphi}{10} . \qquad \text{(EQ 3)}$$

We believe that this result will help immeasurably as researchers push back the boundaries of knowledge throughout the discipline of optics, and throughout physics in general.

Energy Loss in Bouncing Balls

J. K. FINKLEBOTTOM, P. SNERD, AND J. M. W. DOOGIN

ABSTRACT
We discuss how bouncing balls lose energy.

One easily observes that bouncing balls are not perfectly elastic. When a ball bounces, some energy is lost; as a consequence, the ball will never return to its original height, and eventually will come to rest.

This energy loss dissipates as heat (which is nearly undetectable) and sound. Some investigators such as Turpin (2001) suggest that energy is lost when the ball deforms during the process of bouncing. More precisely, the kinetic energy of the falling ball is converted into potential energy in the compressed ball (not unlike a spring), and is re-converted into kinetic energy as the ball bounces upwards.

It is this conversion process which is imperfect. Not all the falling kinetic energy is converted into potential energy, and not all the stored potential energy is converted into kinetic. Thus, on every bounce, an amount of energy, ΔE, is lost to the system.

This elegant theoretical model has some obvious consequences, easily verified by experiment. For example, a ball that strikes the ground with energy E will subsequently undergo $N_B = E/(\Delta E)$ additional bounces. This is in accordance with casual observation: a ball dropped from a higher place will bounce more times.

One can also create a numerical simulation of the bouncing phenomenon. If each "row" is a bounce, and we are given the ball's mass m, its initial downward speed $V_{down}(0)$ and the energy loss ΔE, we can calculate the kinetic energy of the falling ball, the kinetic energy of the rising ball, and the corresponding rising speed $V_{up}(0)$. From that, knowing the acceleration of gravity, we can calculate how long it will take for the ball to land, where we have a new downward speed $V_{down}(1)$, which is equal to $V_{up}(0)$.

Running this simulation, we plot in Figure 1 the time between bounces as a function of the "bounce number"; we see that, as we expect, that time decreases rapidly. As we easily observe with real bouncing balls, the bounces get closer together the further the bouncing progresses.

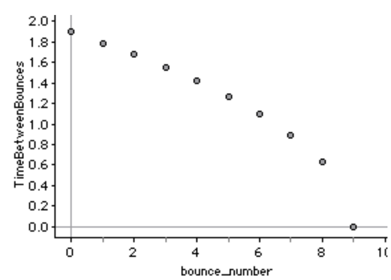


Figure 1: results of the numerical simulation

On the Speed of Rolling Balls

W. L. W. CORPORAL AND A. GRAINYEAR

ABSTRACT
We present data to support a linear relationship between the distance a ball has rolled down a ramp and its speed.

It is a matter of elementary physics that an object moving with constant velocity has a position that increases linearly; and an object undergoing constant straight-line acceleration has a position that increases quadratically. These truths are both embodied in the kinematic equation

$$x = x_0 + v_i t + \frac{1}{2} a t^2 \qquad \text{(EQ 1)}$$

where x is position, x_0 is initial position, v_i is initial speed, a is acceleration, and t is the elapsed time. (If $a = 0$, the formula simplifies into the linear case: constant velocity.)

This relationship holds, at one lower power, for the speed itself: the speed of an object with constant velocity is constant, and the speed of an object undergoing constant acceleration increases (or decreases) linearly.

We test this notion by measuring the (instantaneous) speed of a ball, released from rest, after it has rolled down a ramp over different distances.

Figure 1 shows speed data for a tennis ball rolled down a straight ramp. The distances are the distances along the ramp in centimeters. The figure also shows a least-squares regression line; you can see that the fit is excellent with a value of $r = 1.00$. As you can see, the speed increases linearly, as we predicted. This also makes intuitive sense: the farther up the ramp you release a ball, the faster it is going when it reaches the bottom.

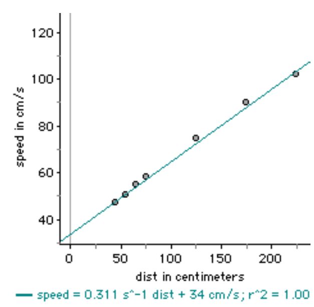


Figure 1: data showing speed from different distances.

The principal parameter in the relationship—the slope of this line —depends on the slope of the ramp, but also on properties of the ball (such as whether it is hollow or not) and other systemic effects (such as rolling friction).

This linear relationship will have many practical applications, notably in highway safety and roller-coaster design.